Wavelet Domain Nonlinear Filtering for Evoked Potential Signal Enhancement

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A wavelet domain nonlinear filtering method for improving the signal-to-noise ratio (SNR) of the evoked potentials (EP) is proposed. The method modifies the selective filtering technique proposed for edge detection in images by Xu *et al.* for the case of signals which require a smooth transition at the edge points. It identifies the significant features of a noisy signal based on the correlation between the scales of its nonorthogonal subband decompositions. The signal transition information from interscale correlation coupled with the change in variance around the identified transition region is used to differentiate between noise and the signal. A nonlinear function such as a Gaussian smoothing function applied around the identified edge in the wavelet domain leads to smoothing in the signal space also. Numerical results obtained by applying the proposed nonlinear filtering method on middle latency responses of auditory evoked potentials show that the method is well suited for signal enhancement applications.

1. INTRODUCTION

Evoked potentials (EP) are the responses of the brain and result by stimulating a sensory pathway. These responses can be measured using surface electrodes placed at specific locations on the scalp. The spontaneous activity of the brain, called electroencephalogram (EEG), is the major source of corrupting noise while measuring the EP signals. The amplitude of the noise is much higher than that of EP response. The EEG noise is predominant in a recorded EP with typical SNR ranging from −5 down to −20 dB. Classical filtering techniques are not suitable because the spectra of EP and the background EEG significantly overlap. The most common method used for recovering the EP signal from the background EEG noise is by synchronous averaging of a number of single sweeps time-locked to external stimuli. Basic assumptiosn of synchronous averaging are that the signal and noise are stationary, uncorrelated random processes, noise has zero mean across the ensemble, and the signal is determinstic. To obtain a satisfactory estimate, a few hundreds to a few thousands of response trials are required to be averaged, depending on the modality of the stimulus and the type of diagnostic test being conducted.

There is believed to be a need for reduction in the time needed to obtain a reliable estimate of the EP for certain applications. In surgeries involving spinal cord, such as scoliosis correction, it will be a boon if the EPs can be obtained with much fewer sweeps. This will facilitate continuous monitoring of the integrity of the neural pathways in the spinal cord. In clinical situations involving short-acting anesthesia, such as electroconvulsive therapy, if one is interested in studying the dynamic changes in neural conduction for those 5- to 10-min periods, one needs EP estimates much faster than is possible through ensemble averaging. There is also interest in studying changes in spinal conduction velocities during yogic postures involving spinal flexions and twists. Figure 1 shows a single sweep auditory evoked potential and an ensemble average of 30 sweeps in which the characteristic components are visible. However, the quality of the extracted signal is not adequate.

To obtain a reliable signal estimate with fewer sweeps, several algorithms have been proposed which claim to improve the signal-to-noise ratio of the averaged evoked potentials. In *a posteriori* Wiener filtering, de Weerd and Martens (*5*) have deduced signal and noise power spectra from the observations to design an optimal filter to be used to improve the signal-to-noise ratio of the averaged EP. To account for the nonstationary components of the EP signal, they used a bank of filters cascaded together to implement time-varying filter structures. Time domain techniques which use *a priori* knowledge of the signal and noise statistics to design an optimal filter transfer function (*8*) and *a posteriori* optimal filter which does not require any such knowledge (*6*) have been attempted to achieve the same goal. These methods vary in their approaches for estimation of signal and noise autocorrelation functions which are used in defining the filter transfer functions. Wavelet domain filtering by Bertrand *et al.* (*4*) attempts to use Wiener filtering in a time–frequency domain using the wavelet spectra. This is much simpler to design and also is invertible. In the absence of knowledge about the noise statistics, transform domain filtering, which takes local statistics into consideration rather than global signal structure, is a better choice. Wavelet decomposition is very well suited for the cases in which signal and noise have overlapping frequency bands. In the wavelet domain, noise attenuation can be optimized locally with a negligible distortion of the signal details. Wavelet domain filtering is well suited for EP signals for the following reasons.

- The significant events in a EP signal are the nonstationary components that appear along with the stationary correlated noise.
- Some local portions of the signal have important and significant components while there are considerable noise components in the same band.
- Temporally varying thresholding with local estimation of noise variance improves the filter performance.
- The content and the accuracy of the diagnostic information may be improved by using selective filtering.

Noise reduction techniques based on wavelet and subband decompositions mostly work on thresholding the wavelet coefficients. The success of these methods depends entirely on the estimated noise variance in wavelet domain which is used

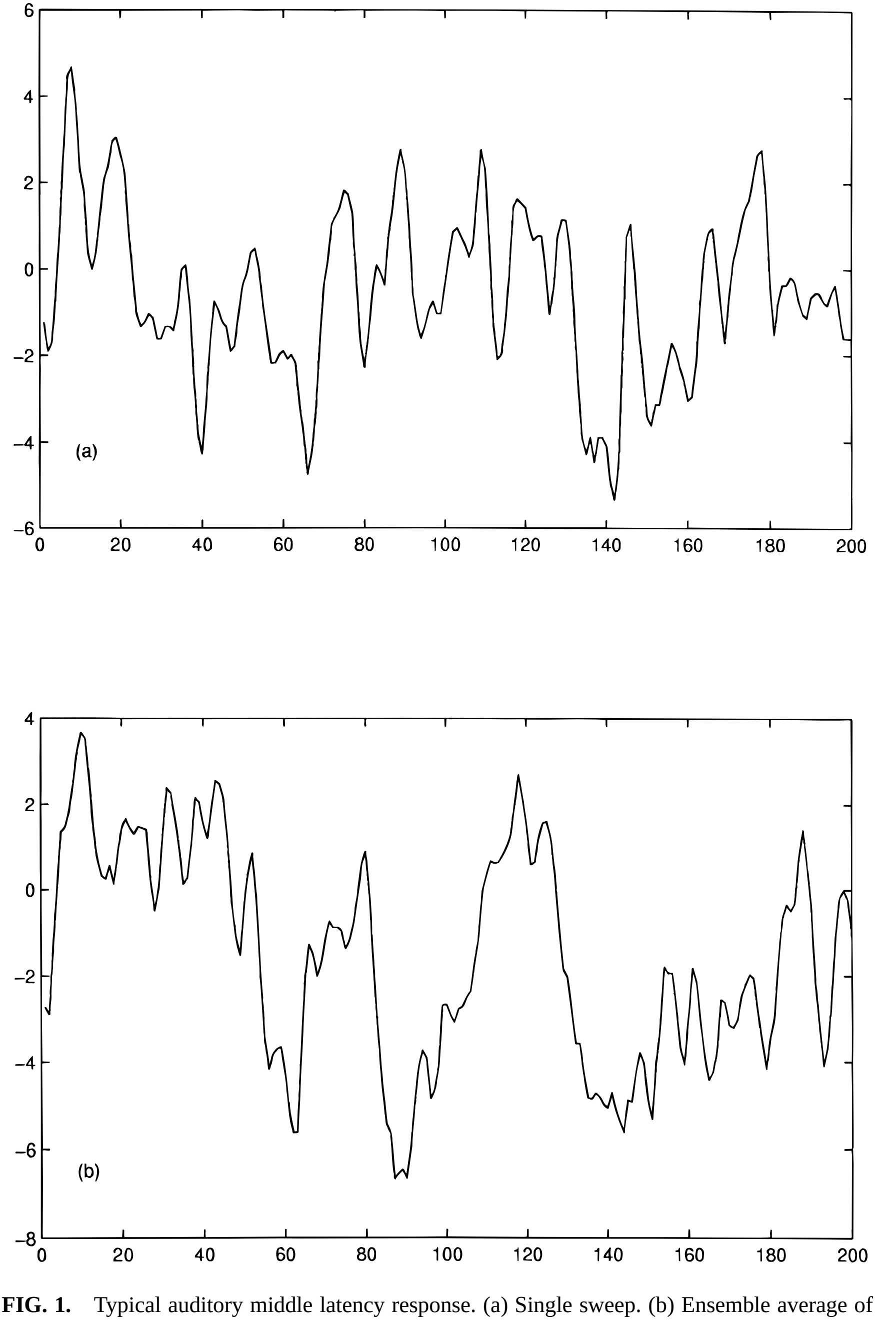

FIG. 1. Typical auditory middle latency response. (a) Single sweep. (b) Ensemble average of 30 sweeps. [*X* axis, Sample No.; *Y* axis, amplitude in relative units.]

to fix the threshold. In the present paper, an attempt is made to improve the SNR of the ensemble average and enhance the significant components of the signal, namely, the peaks and valleys by using nonlinear filtering in a wavelet domain. It adapts the spatially selective filtering technique suggested by Xu *et al.* (*2*) for EP signal enhancement. Their algorithm is modified by using a nonlinear shrinkage function around the identified edges in the wavelet domain to obtain a smooth transition region. We have used an undecimated nonorthogonal discrete wavelet transform (UDWT) using à trous algorithm in which the decomposed signal is correlated across scales. A sharp transition in the signal gives rise to a maximum in several adjacent scales. Hence the correlation between adjacent scales is used to identify the signal transition regions. Instead of retaining just the identified edges, and masking out the rest of the wavelet coefficients, a Gaussian function is placed at the identified edge, scaling wavelet coefficients around the edges. This ensures retention of not only the signal peak information but also the surrounding information. To distinguish and remove transient and spurious noise, the variance of the wavelet coefficients in a window region around the identified transition region is made use of. This filter may be viewed as a low-pass filter that passes selected high-frequency data.

This paper is organized as follows. In Section 2 we briefly explain the orthogonal and nonorthogonal wavelet decomposition using the two popular algorithms, namely, Mallat's pyramidal algorithm and thè a trous algorithm. Selective filtering in wavelet domain and the suggested modification is explained in Section 3. The results obtained using evoked potential data are presented in Section 4 with a discussion and conclusion following in Section 5.

2. WAVELET DECOMPOSITION

The wavelet representation provides a multiresolution/multifrequency description of a signal with localization in both time and frequency. Basically, the transformation amounts to projecting a signal on a family of elementary functions, obtained by translating and dilating a single basic function, $\psi(x)$, called *analyzing wavelet*,

$$\psi_{a,b}(x) = |a|^{-1/2}\, \psi\left(\frac{x - b}{a}\right) \quad (a, b) \in \mathbf{Z}, \qquad [1]$$

where a and b are the scale and the translation parameters, respectively. The definition of the wavelets as dilations of the function $\psi_{a,b}(x)$ means that high-frequency wavelets correspond to $a < 1$ or narrow width, while low-frequency wavelets have $a > 1$ or wider width. The basis functions used in wavelet transforms are locally supported; they are nonzero over the part of the domain represented.

Wavelet transform decomposes a nonstationary signal into a set of multiscale components where each component is relatively more stationary. In discrete wavelet transform (DWT), only a discrete set of scale parameters is used. The multiresolution scheme proposed by Mallat (*3*) is based on a fixed dyadic grid to obtain the basis wavelets. Mallat's multiresolution decomposition and the à trous algorithm are two separate implementations of the discrete wavelet transform. Both algorithms are observed to be special cases of a single filter bank structure.

2.1. Multiresolution Decomposition—Pyramidal Algorithm

In multiresolution decomposition, a signal is decomposed into a set of different frequency channels of constant bandwidth on a logarithmic scale. In this, in addition to the analyzing wavelet function, ψ, a scaling function, ϕ, is introduced. The dilated and translated versions of the scaling function may be obtained as

$$\psi_{j,k}(x) = 2^{-j/2}\phi(2^{-j}x - k). \qquad [2]$$

The space spanned by $\psi_{j,k}$ for a fixed j is denoted by V_j. A function $f(x) \in L^2(\mathcal{R})$ is projected, at each step j, onto the subset V_j ($j \leqq 0$). This projection, $s_{j,k}$, is defined as the scalar product of $f(x)$ with the scaling function $\phi(x)$

$$s_{j,k} = \langle f(x), 2^{-j}\phi(2^{-j}x - k)\rangle. \qquad [3]$$

k and j are the translation and dilation parameters, respectively. The successive approximation spaces, $\ldots \subset V_2 \subset V_1 \subset V_0 \subset V_{-1} \subset V_{-2} \ldots$, each have a resolution 2^j. This results in a decreasing sequence of closed subspace, $V_j \in Z$ which approximate $L^2(\mathcal{R})$. $\phi(x)$ has the following property

$$\frac{1}{2}\phi\left(\frac{x}{2}\right) = \sum_n h(n)\phi(x - n). \qquad [4]$$

The sequence $\{h(k), k \in Z\} \in L^2(\mathcal{R})$ is the impulse response of a low-pass filter. The complementary subspace, W_j is generated by a wavelet $\phi(x)$ with integer translation and dyadic dilation. The projection of $f(x)$ on subspace W_j is defined as

$$d_{j,k} = \langle f(x), 2^{-j}\psi(2^{-j}x - k)\rangle. \qquad [5]$$

The analyzing function $\psi(x)$ has the following property:

$$\frac{1}{2}\psi\left(\frac{x}{2}\right) = \sum_n g(n)\phi(x - n). \qquad [6]$$

The behavior of the computed discrete wavelet transform is governed by the choice of the filters [7]. The filters $h(n)$ and $g(n)$ in pyramidal algorithm are quadrature mirror filters. This decomposition is translation variant because of decimation. It is found to be very useful for coding applications because it removes redundant information.

2.2. Undecimated Wavelet Transform—À Trous Algorithm

In this algorithm, the decomposition is not decimated and only the filters are dilated at each projection. Therefore, each wavelet's scale has N points corresponding to the N data points. For the scale i, these points correspond to 2^i different decompositions obtained with the decimated transform using all the circulant shifts of the signal. These decompositions, each one composed of $N/2^i$ points, are intertwined. This overcomes the dependence of the transform on the position of the input signal. Hence, it is translation invariant unlike the decimated wavelet transform. Figure 2 shows the system block diagram showing the computation of the undecimated wavelet transform. In this algorithm, the sampled data are assumed to be corresponding to zeroeth scale and is denoted as $\{c(0, k)\}$. This can be viewed as a scalar product of a function $f(x)$ with a scaling function $\phi(x)$ which corresponds to a low-pass filter. The subsequent smoothed sequences are given by,

$$c(i, k) = \sum_l h(\mathrm{l})c(i - 1, k + 2^{i-1}l) \qquad [7]$$

and the discrete wavelet transform coefficients are given by the difference between two successive smoothed sequences

$$w(i + 1, k) = c(i + 1, k) - c(i, k). \qquad [8]$$

The coefficients $\{h(k)\}$ are derived from the scaling function $\phi(x)$

$$\frac{1}{2}\phi\left(\frac{x}{2}\right) = \sum_l h(l)\phi(x - l). \qquad [9]$$

The reconstruction is the sum of all wavelet scales and the smoothed signal at the coarsest scale

$$c(0, k) = \sum_{i=1}^{L} w(i, k) + c(L, k) \qquad [10]$$

with L number of decomposition scales.

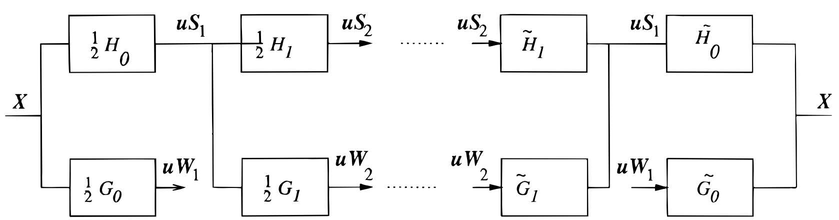

FIG. 2. Undecimated wavelet transform.

The conditions necessary for perfect reconstruction in the case of decimated wavelet transform need not be met in this algorithm because the coefficients are not downsampled. The B_3-spline scaling function is used as the base wavelet in our calculations. The associated filter $H(z)$ is given by,

$$H(z) = \frac{1}{16}z^{-2} + \frac{1}{4}z^{-1} + \frac{3}{8} + \frac{1}{4}z + \frac{1}{16}z^2. \qquad [11]$$

In applications where correlation between scales is used, thè a trous algorithm using nonorthogonal wavelets is found to be well suited. Interscale correlation information in an undecimated wavelet transform (UDWT) has been used for edge detection and also for denoising (*1, 2*). In the case of images, it was found that an edge occurs at a position where there are maxima in the nonorthogonal wavelet transform at several adjacent scales. This property is used to identify and enhance significant component regions of the EP signal in a nonlinear fashion as explained in the following section.

3. NONLINEAR FILTERING IN A WAVELET DOMAIN

A simple and efficient technique for edge detection in a wavelet domain for images has been suggested by Xu *et al.* (*2*). They have proposed a spatially selective wavelet filtering approach based on interscale correlation using UDWT. Direct spatial correlation between adjacent scales, $\mathrm{corr}_2(m, n)$ is defined to be

$$\mathrm{corr}_2(m, n) = w(m, n)w(m+1, n) \quad n = 1, 2, \ldots, N. \qquad [12]$$

Their method is based on the fact that sharp edges have large amplitude over many wavelet scales, and noise dies out swiftly with increasing scale. A large spatial correlation between the scales is used to detect edges. A binary mask is used to retain or discard wavelet coefficients. We found that using a binary mask for selecting the wavelet coefficients at each scale gives rise to oscillations or visual artifacts at the point of discontinuity. Hence, we attempted to optimize noise attenuation locally with a negligible distortion of the signal details using a Gaussian mask rather than a binary mask. This helps in avoiding Gibb's oscillations or visual artifacts around the identified significant features. The proposed filtering method works in three stages. Initially, it identifies the significant features by finding the regions where correlation between adjacent wavelet scales is high. Then, as a second step, it uses the change in variance in the local neighborhood of the identified edge to exclude detection of the correlated noise edges. Finally, a Gaussian shrinkage function centered at the edge point scales the wavelet coefficients to enhance the nonstationary signal components.

An edge is identified if $|\mathrm{corr}_2(m, n)| > |w(m, n)|$. To differentiate a signal edge from a sharp correlated noise edge, we define a small region in each scale, m, before and after the identified edge location, n, and compare the variance in both the regions. Defining $\sigma_1(n) = \mathrm{var}(\mathrm{win1} = w(m, n-k{:}\ n-1))$, and $\sigma_2(n) = \mathrm{var}(\mathrm{win2} = w(m, n+1{:}\ n+k+1))$, both windows of width k, in scale m,

$$
\begin{aligned}
&\text{if} \quad \text{abs}(\sigma_1(n) - \sigma_2(n)) < \epsilon \\
&\qquad \text{discard the edge} \quad \text{(indicates noise edge)} \\
&\text{else} \\
&\qquad \text{retain the edge} \quad \text{(indicates signal edge)} \\
&\text{end}
\end{aligned}
$$

where $\epsilon \in R$ is fixed as a small fraction of $\sigma_1(n)$ and is positive. The absence of edges or other significant features in a localized region of the signal allows the noisy background to be removed, thereby equating the wavelet coefficients in that region to zero. The Gaussian shrinkage function $\eta(w(m,n))$ centered at the identified edge k, with a standard deviation $p(n) = 0.1\sigma_1(n)$ is defined as

$$\eta(w(m, n)) = \frac{1}{p(n)\sqrt{2\pi}} e^{-\left(\frac{(w(m, n) - k)^2}{p^2(n)}\right)}. \qquad [13]$$

The wavelet filter has feature-sensitive selectivity in passing high-frequency data at low SNR. The small-scale data are passed at positions where the correlation is large and suppressed if the correlation is small.

4. EXPERIMENTAL RESULTS

The proposed filtering method is applied on the clinically recorded middle latency auditory responses (MLR) and brain stem auditory evoked potentials (BAEP). Components occurring within the first 10 ms of the application of stimulus are called the early components or BAEP and components occurring between 10 and 60 ms are termed middle latency components. The signals are recorded at the vertex with a reference on the ipsilateral mastoid and are obtained with repetitive application of auditory clicks of 0.5 ms at 100 dB above auditory threshold at a rate of 10 Hz. The subject is asked to keep his eyes closed throughout the session to minimize ocular artifacts. MLR responses have four characteristic peaks, as shown in Fig. 1b. However, because of the low SNR, the characteristic peaks are not visible in single responses. Its decomposition into four scales with a B_3-spline scaling function using nonorthogonal UDWT as per the à trous algorithm is shown in Fig. 3. From the correlation between adjacent scales which is also shown in the same figure, it can be seen that the significant features in the original signal such as sharp peaks and valleys appear stronger in the correlation domain than in the wavelet domain. By using the spatial selective filtering algorithm suggested in (*2*), a binary mask to be applied on the different wavelet scales is obtained. To avoid detection of spurious noise as a significant feature, the variance in a small window region before the identified edge is compared with the variance in a region after the edge. Because the noise is assumed to be a stationary process, this measure eliminates detection of transient noise edges mistakenly as significant features. The masks obtained after using the variance check along with the Gaussian shrinkage function used in place of the binary mask to smooth the surrounding region

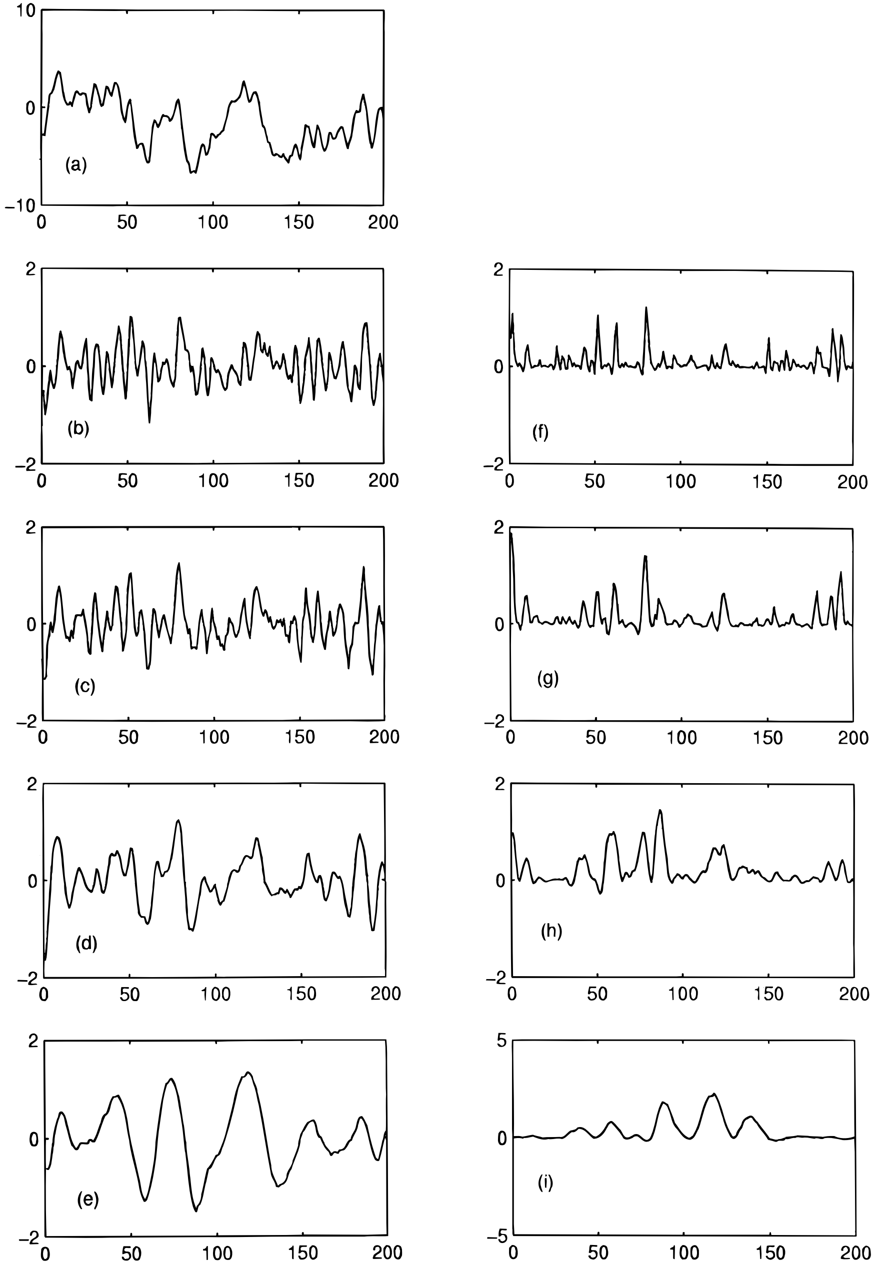

FIG. 3. Nonorthogonal decomposition of the signal and the correlation between its adjacent scales. (a) The signal. (b–e) Components at scales 1, 2, 3, and 4, respectively. (f–i) Correlation between scales 0 and 1, 1 and 2, 2 and 3, and 3 and 4, respectively. [*X* axis in all cases, Sample No.; *Y* axis in b–e, amplitude in relative units; in f–i, correlation in relative units.]

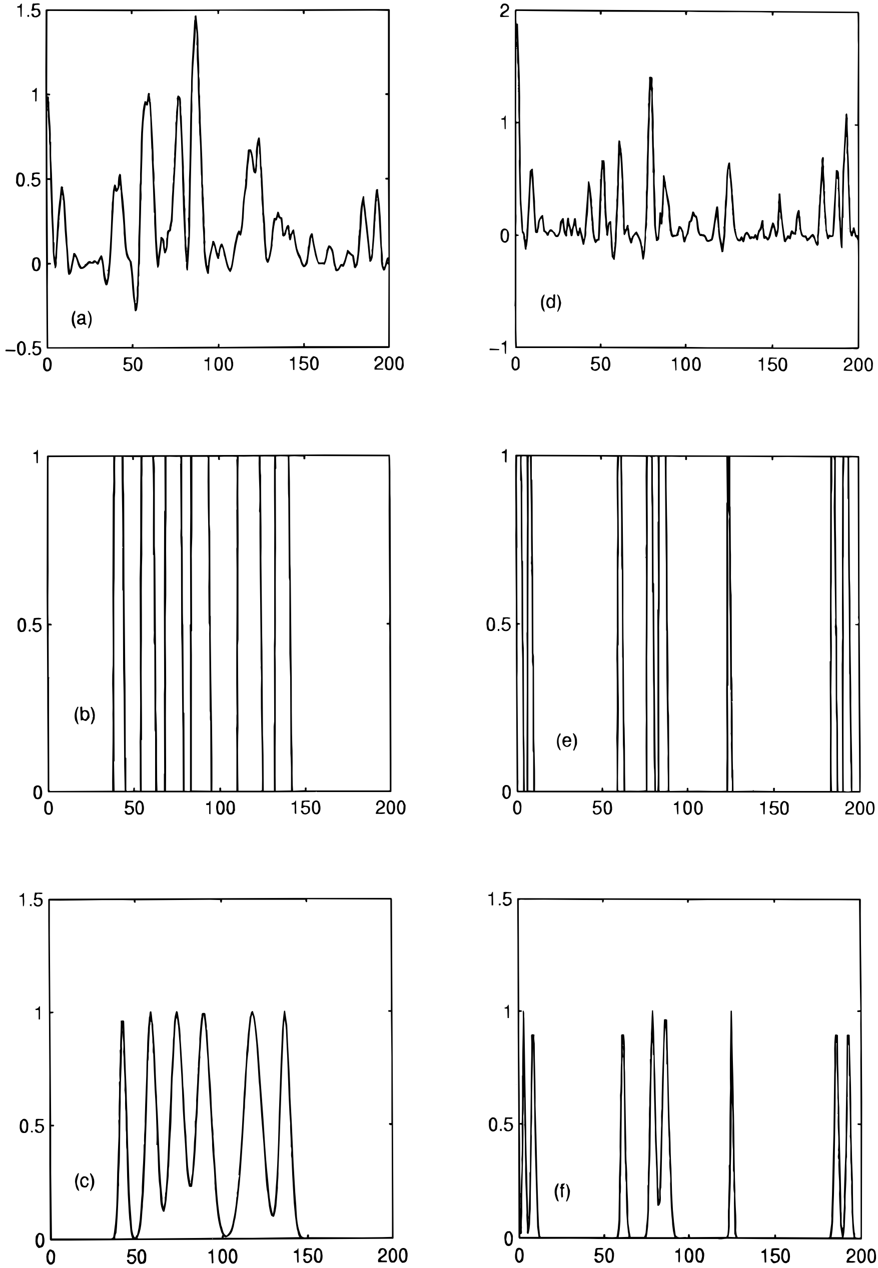

FIG. 4. Wavelet coefficients retained using binary mask and Gaussian mask. (a) Correlation between scales 2 and 3, (b) Binary mask obtained using selective filtering. (c) Gaussian mask using our method. (d–f) Corresponding figures for correlation between scales 3 and 4. [*X* axis in all cases, Sample No.; *Y* axis in a and d, correlation in relative units; in b, c, e, and f), magnitude in relative units.

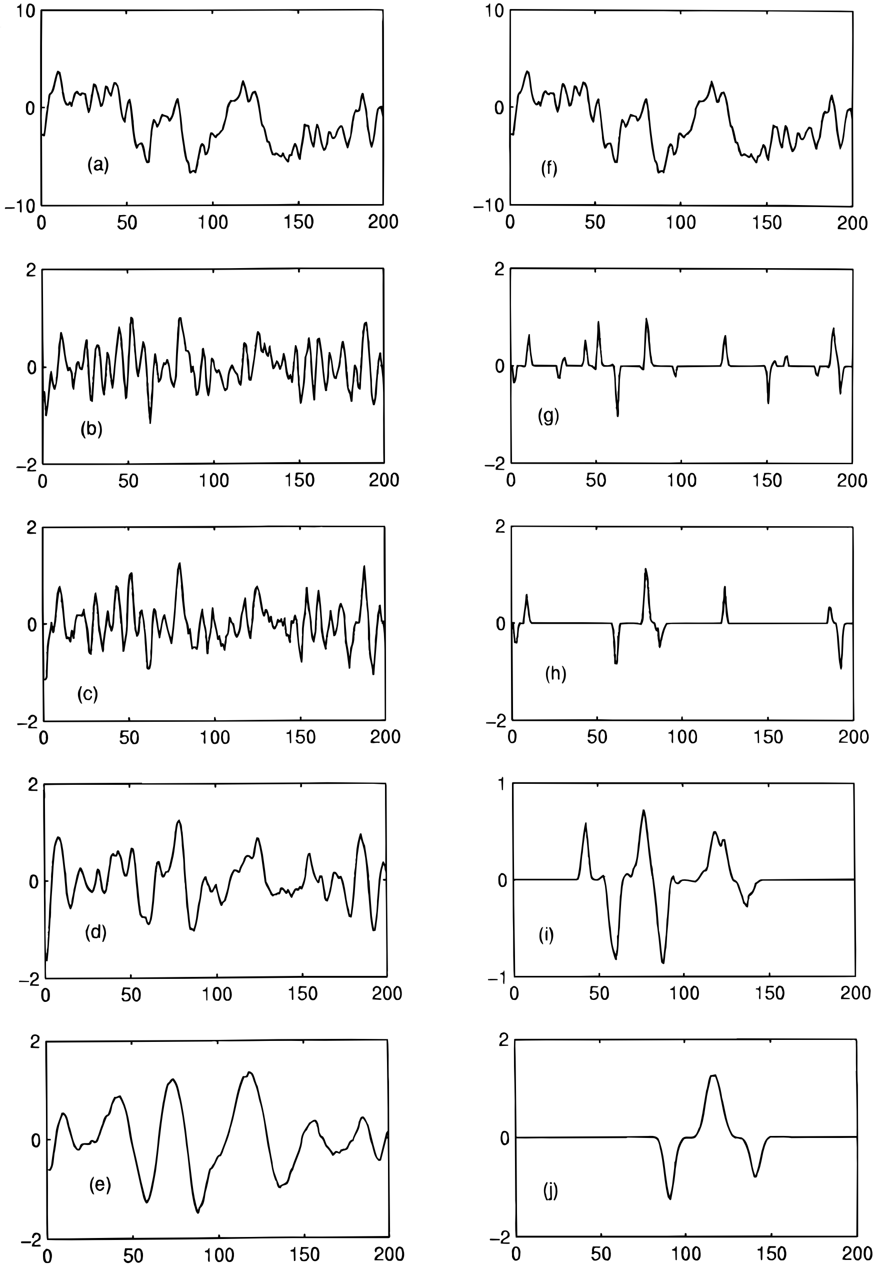

FIG. 5. Wavelet coefficients using UDWT at different scales before and after nonlinear filtering using Gaussian functions. (a, f) Original signal. (b–e) The wavelet decompositions at scales 1, 2, 3, and 4, respectively. (g–j) The corresponding filtered decompositions. [X axis, Sample No.; *Y*-axis; amplitude in relative units.]

of a significant feature are shown in Figs. 4c and 4f for two different correlation levels, namely, $corr_2$ (2, 3) and $corr_2$ (3, 4). Figure 5 shows the decomposition of the MLR using UDWT into different scales and the modified wavelet coefficients using the nonlinear filtering method.

The signal obtained by the proposed method is compared with the one obtained using the binary mask in Fig. 6. It can be seen that the small variations which are present while using binary mask filtering are removed well with the nonlinear filtering which considers the variance around the identified edge region to retain

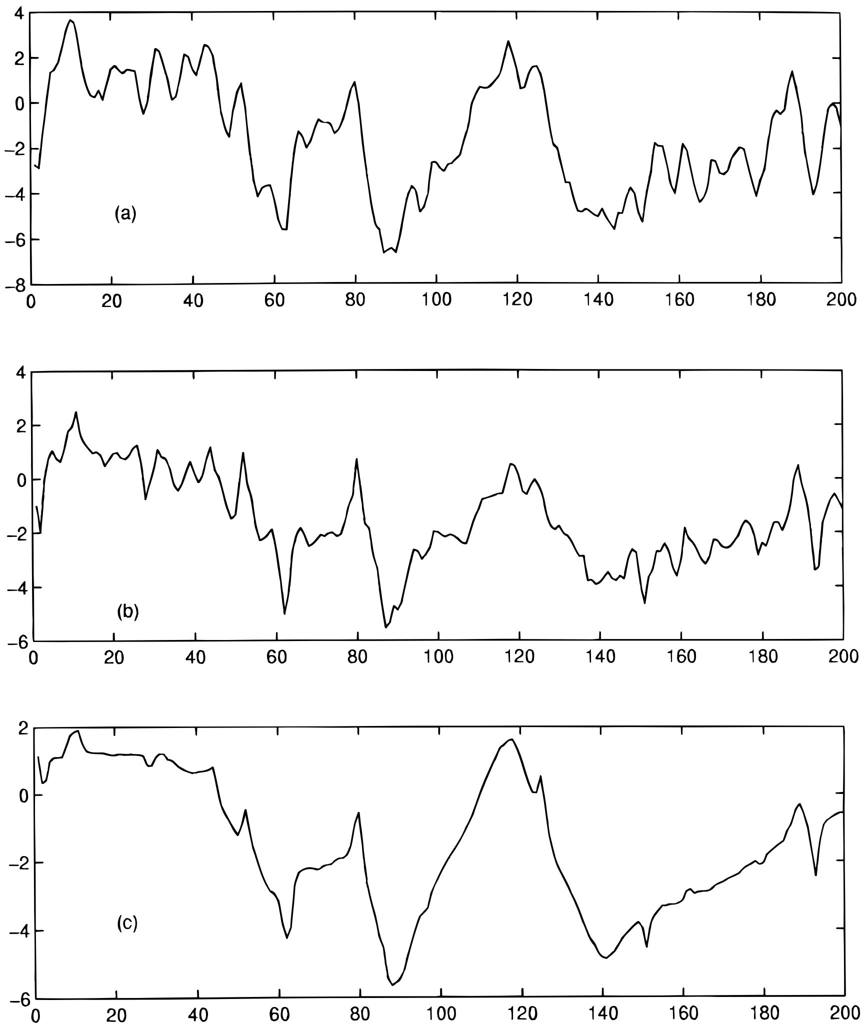

FIG. 6. Comparison of outputs obtained using binary and Gaussian masks. (a) Middle latency AEP shown in Fig. 1a. (b) Filtered output obtained using binary masks. (c) Output obtained using Gaussian masks. [*X* axis, Sample No.; *Y* axis, amplitude in relative units.]

or suppress the wavelet coefficients. The technique applied on a typical middle latency visual evoked potential is shown in Fig. 7. In this figure too, it can be observed that the nonlinear filtering around the identified edge helps in enhancing the edge information and smoothing the insignificant small variations around the edges.

Comparison of the proposed filtering technique with the conventional ensemble average (EA) of various ensemble lengths is carried out, since ensemble averaging

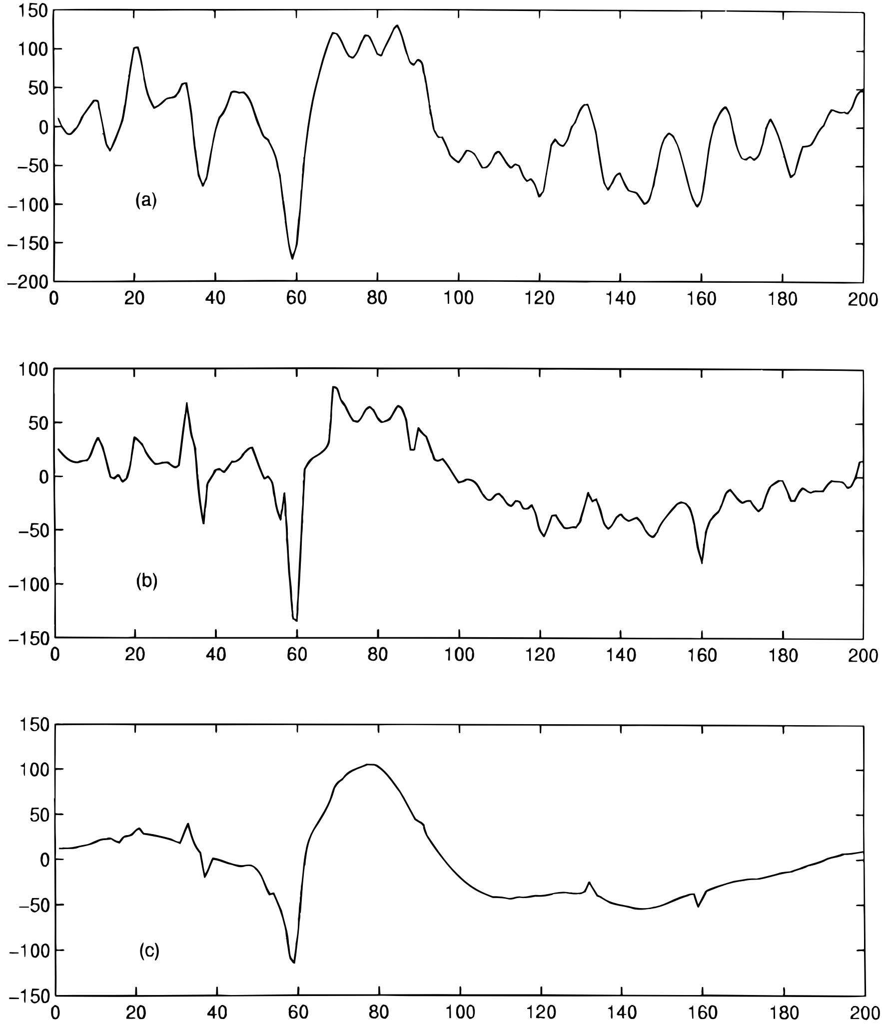

FIG. 7. Comparison of filtered outputs using binary and Gaussian masks. (a) Middle latency visual evoked potential. (b) Output obtained using binary filtering masks. (c) Output obtained using Gaussian filtering masks. [*X* axis, Sample No.; *Y* axis, amplitude in relative units.]

is the most widely used method in clinical practice. In Fig. 8 we present the results obtained for the MLR data shown in Fig. 1b using the proposed filtering technique for different ensemble lengths. It can be seen that the method results in enhanced and smoother component peaks. In the case of BAEP signals, because the SNR of these signals is very low, for clinical diagnosis, at least 512 sweeps are averaged to estimate these signals. In Fig. 9, ensemble averaged BAEP signals at different ensemble lengths are shown along with the processed signals. It can be noted that for any ensemble length that is considered, the filtered signal results in an improved estimate suppressing the wriggle in the component peaks, thereby facilitating accurate measurement of diagnostically important measures such as latencies and amplitude. It may be observed from both Figs. 8 and 9 that changes in the EA are also reflected in the processed output.

Because the computation of wavelet is merely a multistage filtering operation, it is neither computationally complex nor expensive. The interstimulus interval is shortest for the ABER, which is 100 ms. Of this 75 ms is available for computation after the acquisition of individual sweep of 25 ms. Computation of our algorithm takes only 0.853 ms using a Pentium 300 MHz processor under Windows NT

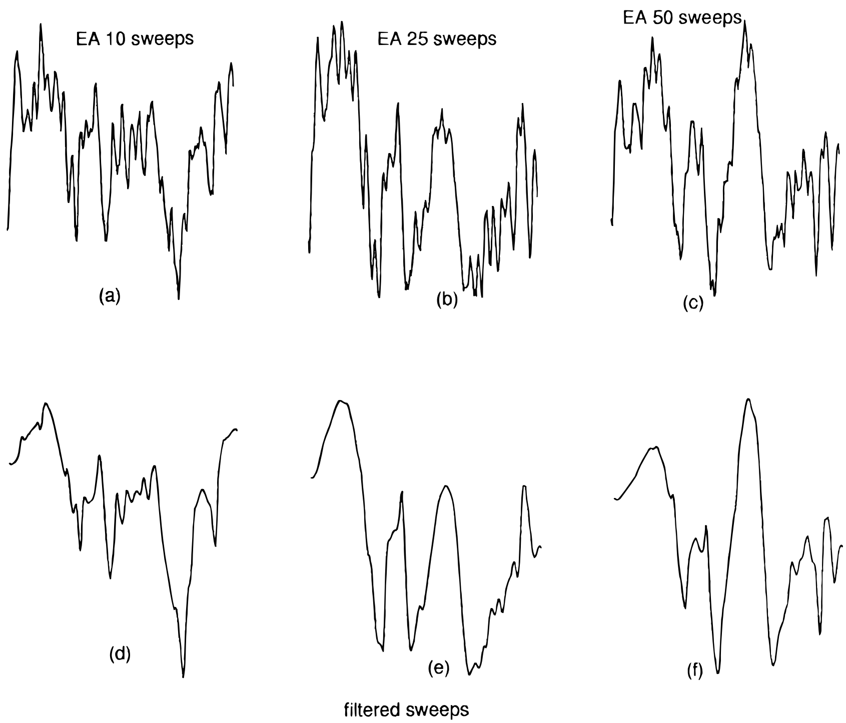

FIG. 8. Comparison of filtered outputs with EA at different ensemble lengths for MLR data. (a–c) The ensemble averages. (d–f) The corresponding filtered sweeps.

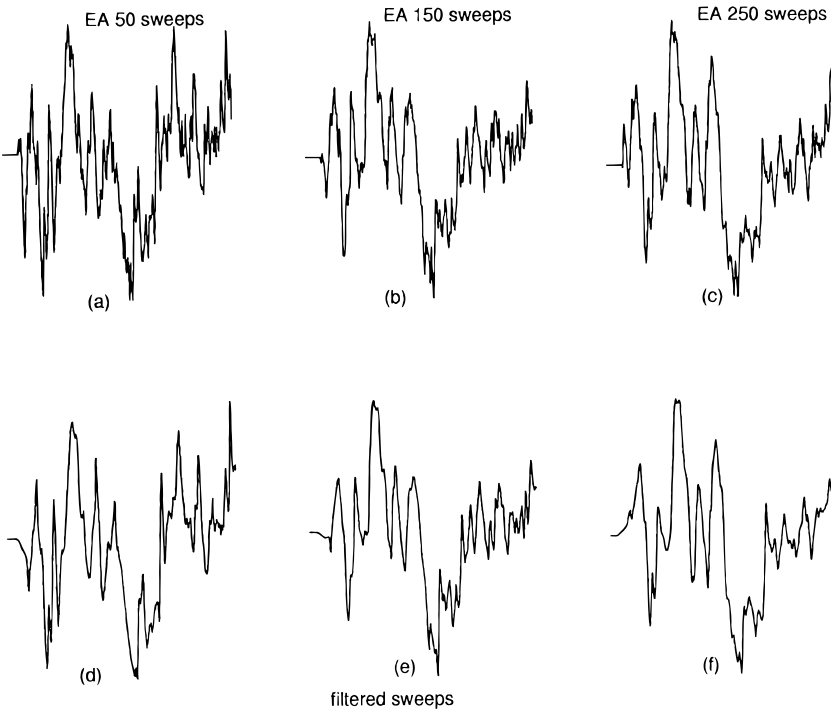

FIG. 9. Comparison of filtered outputs with EA at different ensemble lengths for AEP data. (a–c) The ensemble averages. (d–f) The corresponding filtered sweeps.

Operating System in MATLAB environment. With the inclusion of ensemble averaging, total computation time is around 1 ms out of the available 75 ms. Thus, the algorithm can be integrated into any EP system.

5. CONCLUSION

The EP signal does not have very sharp edges, unlike the case of images. This calls for an approach different from merely hard thresholding of wavelet coefficients at edge locations. The wavelet domain nonlinear filter is superior because of its edge and feature-sensitive selectivity in passing and scaling the wavelet coefficients. The small variations around the identified edge are suppressed to enhance the signal component regions. Temporally varying the threshold with local estimation of noise variance can account for slow transition regions better. The features that were the same size as the noise are distinguished by using the change in variance around the identified edge. By using this additional measure, false detection of sharp noisy edges can be avoided.

When the signal is corrupted by correlated noise which cannot be separated in either frequency or time domain, a signal based local filtering of the noisy signal in transform domain is a promising solution for improving the balance between detail preservation and noise attenuation. We have used a nonlinear filtering method in wavelet domain for enhancement of evoked potentials corrupted by stationary correlated noise. Hard thresholding the regions where correlation is high between adjacent scales of nonorthogonal wavelet tranform causes visual artifacts and Gibbs oscillations at the significant regions. A nonlinear Gaussian function at the transition region is found to be better suited for signals whose nonstationary regions are not too sharp unlike edges in images. The proposed method can be used in cases where the signal is nonstationary and a proper noise model is not available and is most suitable as a postprocessing tool for EP signal enhancement.

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